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Two-Equation Model of W. C. Reynolds for Isotropic Turbulence

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In the two-equation model of W. C. Reynolds (1974, 1976) for isotropic turbulence, a closure assumption is made for the rate of change of the isotropic dissipation. We computed in our previous paper the triple velocity correlation function from the Kármán-Howarth equation by using experimentally determined double velocity correlation function and by introducing a parameter that represents the time dependence of the turbulence structure. It is found that our parameter is equivalent to the Reynolds' empirical constant in his closure formulation. We examine here the validity of the Reynolds' formula based on our experimental results.

REYNOLDS' TWO-EQUATION MODEL FOR ISOTROPIC TURBULENCE

The dynamical equations for the turbulence energy q^2 and the isotropic dissipation D can be derived from the Navier-Stokes equations by simple manipulations. For homogeneous isotropic turbulence the equations are reduced to

$$dq^2/dt = -2D, \qquad dD/dt = -W \tag{1}$$

with $q^2 = \overline{u_i u_i}$, $D = \nu\overline{u_{i,j}u_{i,j}}$ and $W = 2\nu\overline{u_{i,j}u_{j,k}u_{k,i}} + 2\nu^2\overline{u_{i,jj}u_{i,kk}}$, where t is the time, ν is the kinematic viscosity, u_i is the fluctuating velocity component, and the subscript after comma denotes partial differentiation with respect to the Cartesian coordinate: $u_{i,j} = \partial u_i/\partial x_j$, $u_{i,jj} = \partial^2 u_i/\partial x_j^2$, etc. On dimensional grounds Reynolds (1974) makes the closure assumption

$$W = c_7 D^2/q^2 \tag{2}$$

to obtain the decay formulae

$$q^2 = q_0^2(1 + t/a)^{-n}, D = D_0(1 + t/a)^{-n-1},$$
$$a = nq_0^2/2D_0, n = 2/(c_7 - 2) \tag{3}$$

where q_0^2 and D_0 are the initial values for $t = 0$.

On the basis of the experimental results of Comte-Bellot and Corrsin (1966) and the suggestion of Lumley and Khajeh-Nouri (1974), Reynolds (1976) proposes $c_7 = 11/3$ for large values of the turbulence Reynolds number $R_\lambda = u'\lambda/\nu$, where u' is the rms value of fluctuating velocity and λ is the lateral microscale defined by

$$\lambda^2 = 5\nu q^2/D \tag{4}$$

In the final period of decay, where R_λ is very small, the inertia terms are unimportant and $c_7 = 14/5$ is valid. Reynolds thus proposes

$$c_7 = \frac{11}{3} - \frac{13}{15}\exp[-(R_\lambda^2/20)^2] \tag{5}$$

EXPERIMENTAL RESULTS OF SATO

In homogenous isotropic turbulence the double and triple velocity correlation functions, $f(r,t)$ and $k(r,t)$, for two points separated by a distance r at time t, are related each other by the Kármán-Howarth equation

$$\frac{\partial}{\partial t}(u'^2 f) = u'^3\frac{1}{r^4}\frac{\partial}{\partial r}(r^4 k) + 2\nu u'^2\frac{1}{r^4}\frac{\partial}{\partial r}\left(r^4\frac{\partial f}{\partial r}\right) \tag{6}$$

Measurements were made by the present authors (Sato 1980; Sato et al., 1983) on the double correlation $f(r,t)$ as a function of $\psi = r/\sqrt{2}\lambda$ and R_λ. In terms of the experimentally determined double correlation function the authors calculated the triple correlation function $k(r,t)$ by equation

$$k(\psi,R_\lambda,I_\lambda) = -\frac{\sqrt{2}}{R_\lambda}\left\{2\psi f + \frac{\partial f}{\partial \psi} + \frac{I_\lambda - 2}{\psi^4}\int_0^\psi \psi^5\frac{\partial f}{\partial \psi}d\psi\right\} - \frac{\sqrt{2}\,(5 - I_\lambda)}{\psi^4}\int_0^\psi \psi^4\frac{\partial f}{\partial R_\lambda}d\psi \tag{7}$$

where

$$I_\lambda = (1/2\nu)(d\lambda^2/dt) \tag{8}$$

is the parameter representing the time dependence of the turbulence structure. Since $I_\lambda = -5 - (\lambda^2/2\nu D)(dD/dt)$, it would be possible to determine I_λ by direct measurements, but no reliable result has yet been obtained. Consequently, I_λ has been treated as an empirical parameter. By representing the decay of turbulence energy behind a grid of mesh length M in a uniform stream of velocity $\overline{U}$ in the form

$$u'^2/\overline{U}^2 = B_1\xi^{-m}(1 + B_2\xi)^{m-5/2} \tag{9}$$

where $\xi = x/M$ is the nondimensional distance from the grid, and with the aid of the Taylor's hypothesis of the frozen field, the authors obtained

TABLE 1. MEASUREMENTS OF DECAY OF ENERGY FOR GRID-GENERATED TURBULENCE

Author	R_M ($\times 10^{-3}$)	$\bar{U}$ m/s	Grid M, mm	M/d	Type*	B_1 ($\times 10^2$)	β ($\times 10^{-2}$)	Symbol in Fig. 1
Batchelor et al. (1948)	0.65	6.2	1.6	5.3	Rd.	2.8	0.44	△
Comte-Bellot et al. (1966)	34.0	20.0	25.4	5.3	Rd.	4.5	37.0	▽
Uberoi and Wallis (1967)	26.0	15.2	25.4	4.0	Rd.	4.0	24.0	○
	26.0	15.2	25.4	8.0	Id.	2.7	17.0	⊖
	26.0	15.2	25.4	—	Hc.	7.8	49.0	●
Van Atta and Chen (1968)	25.6	15.7	25.4	5.3	Rd.	2.6	16.0	◇
Tavoularis et al. (1978)	0.475	4.0	1.27	5.2	Rd.	0.35	0.04	▼
Sato (1980)	14.0	5.4	36.0	4.5	Rd.	3.2	11.0	□
	6.6	6.3	15.0	3.0	Rd.	5.7	9.3	■

* Square mesh grid: Rd. = round rods, Id. = inclined rods, Hc. = honeycomb.

$$I_\lambda = 2 + \left(\frac{5}{m} - 2\right)\left(1 + \frac{5}{2m}B_2\xi\right)^{-2} \quad (10)$$

$$R_\lambda^2 = \frac{10}{m}B_1R_M\xi^{1-m}(1 + B_2\xi)^{m-3/2}\left(1 + \frac{5}{2m}B_2\xi\right)^{-1} \quad (11)$$

where $R_M = \bar{U}M/\nu$, $m = 6/5$, $B_2 = 0.001$, and B_1 is a constant depending on the initial conditions including the geometry of the grid as shown in Table 1. Figure 1 shows that the decay of turbulence energy downstream of grid can be expressed by Eq. 9.

EVALUATION OF REYNOLDS' FORMULA

Examination of Eqs. 1, 2, 4, and 8 reveals that our parameter I_λ is equivalent to Reynolds' constant c_7, namely

$$c_7 = \frac{2}{5}I_\lambda + 2 \quad (12)$$

Reynolds suggests to approximate c_7 as a function of R_λ, and it appears desirable to examine its validity based on our data. Elimination of ξ from Eqs. 10 and 11 leads to

$$R_\lambda^2 = \frac{10}{m}B_1R_M\left(\frac{2m}{5B_2}\alpha\right)^{1-m}\left(1 + \frac{2m}{5}\alpha\right)^{m-3/2}(\alpha+1)^{-1},$$

$$\alpha = \sqrt{(5/m-2)/(I_\lambda - 2)} - 1 \quad (13)$$

By using Eq. 13 with the aid of Eq. 12, it is possible to obtain numerically the relation between c_7 and R_λ, which may be approximately expressed in the form analogous to Reynolds' Eq. 5

$$c_7 = \frac{11}{3} - \frac{13}{15}\exp[-(R_\lambda^2/\beta)^{1.56}] \quad (14)$$

with $\beta = 2.44B_1R_M$. Figure 2 shows the comparison of Eqs. 5 and 14. This result indicates that the Reynolds' formula represents a good average of our experimental data, although the initial condition given by the constant β is to be taken into account for a more reliable prediction.

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NOTATION

a	= constant used in Eq. 3
B_1, B_2	= constants used in Eq. 9
c_7	= W. C. Reynolds' constant used in Eq. 2
D	= isotropic dissipation of turbulence energy
f	= double velocity correlation function
I_λ	= parameter defined by Eq. 8
k	= triple velocity correlation function
M	= mesh length of grid
m	= exponent used in Eq. 9
n	= exponent of turbulence decay
q^2	= turbulence energy
R_M	= $\bar{U}M/\nu$, Reynolds number based on mesh length
R_λ	= $u'\lambda/\nu$, turbulence Reynolds number
r	= distance between two points
t	= time
U	= velocity of main flow
u	= fluctuation velocity
W	= scalar for a closure assumption
x	= distance from grid

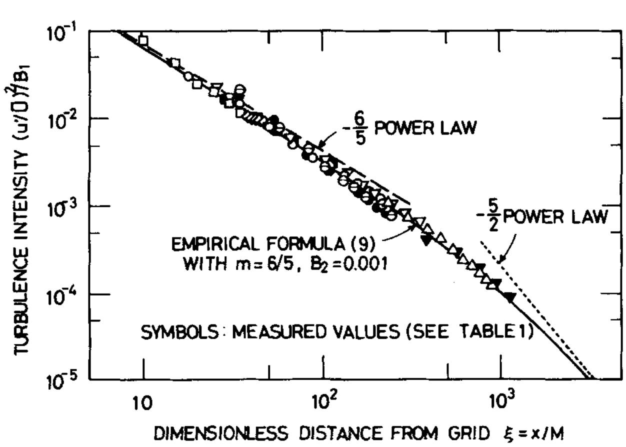

Figure 1. Decay of energy for grid-generated turbulence.

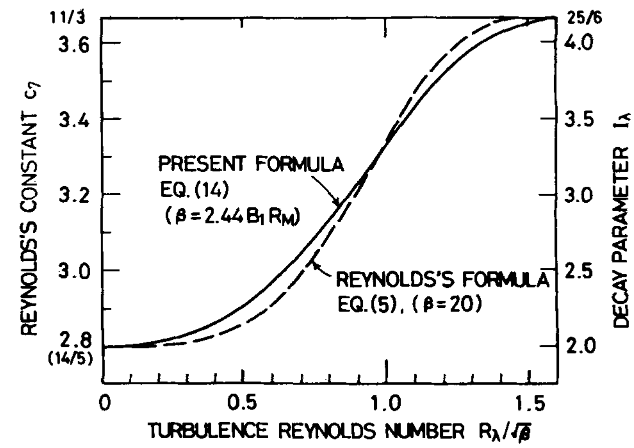

Figure 2. Evaluation of C_7 and I_λ as functions of turbulence Reynolds number R_λ.

Greek Letters

β = empirical constant used in Eq. 13
λ = lateral microscale of turbulence
ν = kinematic viscosity
ξ = x/M, dimensionless distance from grid
ψ = $r/\sqrt{2}\lambda$, dimensionless distance between two points

Subscripts and Superscripts

0 = initial value
, = partial differentiation; $u_{i,j} = \partial u_i/\partial x_j$, $u_{i,jj} = \partial^2 u_i/\partial x_j^2$, *etc.*
i,j,k = Cartesian coordinate components
$-$ = time-smoothed value
= root-mean-square value

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Heat Transfer to a Laminar Flow Fluid in a Circular Tube

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The Graetz (1883, 1885) problem involved the finding of the temperature profile in a fully-developed laminar flow of fluid inside a circular tube. In this communication, we present a general analytical solution in closed form via the method of variable transformation. Also theoretical expressions of Nusselt number (arithmetic mean and logarithmic mean) as a function of Graetz number were obtained.

GRAETZ PROBLEM

The governing equation for the Graetz problem may be obtained from an energy balance in cylindrical coordinates. For a fluid with constant physical properties, neglecting axial conduction, and at steady state, the resulting partial differential equation in the dimensionless form is:

$$(1-\xi^2)\frac{\partial\theta}{\partial\zeta} = \frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) \tag{1}$$

with boundary conditions:

1. at $\xi = 0$, θ is finite,
2. at $\xi = 1$, $\theta = 0$,
3. at $\xi = 0$, $\theta = 1$,

with

$$\theta = \frac{T_w - T}{T_w - T_o}, \quad \xi = \frac{r}{r_1} \quad \text{and} \quad \zeta = \frac{kz}{\rho c_p v_{\max} r_1^2}$$

By the method of separation of variables, we let

$$\theta = Z(\zeta)R(\xi) \tag{2}$$

Equation 1 may be decomposed to the following two ordinary differential equations,

$$\frac{dZ}{Z} = -\beta^2\zeta \tag{3}$$

$$\xi\frac{d^2R}{d\xi^2} + \frac{dR}{d\xi} + \beta^2\xi(1-\xi^2)R = 0 \tag{4}$$

where β^2 is a positive, real number and constitutes an eigenvalue of the system.

The solution of Eq. 3 is

$$Z = c_1 e^{-\beta^2\zeta} \tag{5}$$

where c_1 is an arbitrary constant.

To solve Eq. 4, the following transformations of both dependent and independent variables are performed:

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